Mathematics Knowledge for Teaching: Evidence from Lesson Study

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In this study we take up the issue of teacher knowledge by characterising the quality of one teacher's Mathematics Knowledge for Teaching (Ball, Thames and Phelps, 2008) by examining her Pedagogical Content Knowledge (PCK) that was accessed during a Lesson Study observation in a Chinese primary school. Results show that the participant's PCK was robust in the way she weaved knowledge of content and her students. We explore implications for the development and sharing of this knowledge in the context of her further involvement in our Lesson Study.

Introduction

Mathematics learning should be supported by experiences where students play an active role in the construction of meaning and concept development. While there is little disagreement with the above view, we cannot ignore the role of teachers in developing and using such learning experiences in the classroom. But what are some of the key factors that could have potentially significant impact on the belief and ability of teacher to enact such classroom experiences? This question has been of concern for both researchers and stakeholders in the provision of quality mathematics teaching.

In this context, teacher knowledge continues to be the subject of inquiry (Hill, Rowan, & Ball, 2005; Forrester & Chinnappan 2011; Murphy, 2011). Ball et al. (2005) have been investigating the dimensions and quality of knowledge that teachers need in order to function effectively in the mathematics classroom. This stream of research has yielded evidence for the broad dimension of knowledge called *Mathematics Knowledge for Teaching* (MKT). However, as acknowledge by Ball et al. (2008), such evidences are limited due the tasks that were used in making judgements about MKT. It was argued that research need to widen the contexts against which MKT could be examined more reliably. More specifically, Ball et al. (2008), suggested that research need to examine the quality of MKT in classroom practice.

In the present study, we take up the above issue by investigating the MKT of a primary school teacher as she participated in the Lesson Study (LS) in a Chinese primary school. MKT is multidimensional but our focus was on Pedagogical Content Knowledge (PCK) that was accessed by the participating teacher during the course of the LS. Thus, PCK is seen as a subset of MKT as argued by Ball et al. (2008). A LS is generally driven by a group of teachers with minimal intervention from the researcher. It has been suggested that LSs should be used to examine teacher professional development. For example, Olson, White and Sparrow (2011:56) commented that,

> 'more research is needed to describe how lesson study and other types of professional development supports sustained teacher change and the impact of new pedagogies supported by professional development on student learning'.

Our study has been aimed at answering the research question: What is the nature of PCK as activated during the course of a Lesson Study in primary mathematics?

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Relevant Literature

Lesson Study

In the present study we set out to examine the above issue in the context of Lesson Study. Recent research in mathematics teaching has identified LS as a rich context in which to examine teacher's CK and PCK (Lim 2007; Tepylo & Moss, 2011; Meyer & Wilkerson, 2011). Lesson Studies have generated considerable interest among teachers and mathematics educators. One can advance at least two reasons as to why Lesson Studies have carved out a niche area for mathematics teachers and researchers. Firstly, LS are situated in day-today classroom teaching-learning contexts and is largely initiated and driven by teachers as a professional development activity. Thus LS provide a level of authenticity about teacher knowledge that is not available for access in other contexts such as clinical interviews. Secondly, while LS is conducted as a professional development exercise, it does involve the use of research methodology including design and data analysis.

A key feature of LS is teacher collaboration where individual work as individuals and in groups. Knowledge grows out of this interaction among colleagues and LS provides multiple opportunities for teachers to access and modify their knowledge of mathematics and that of their children (Lewis, Perry and Hurd, 2009). Thus, the discourse between the teachers and students before, during and after lessons generate rich source of data that can be used to gain insight into not only the classroom culture of learning but also changes in their knowledge and understandings.

Theoretical Framework

In examining teacher knowledge that was activated during the course of our Lesson Study, we aimed to characterise the knowledge in terms of a number of dimensions relevant to teachers' classroom practice and the profession of teaching. In this regard, we were guided by the following model of teacher knowledge for teaching mathematics that was advanced by Hill, Ball and Schilling (2008: 174). Four dimensions of their framework are shown in Figure 1. Common Content Knowledge (CCK): Mathematical knowledge and skill possessed by a well-educated adult; Specialised Content Knowledge (SCK): Knowledge of how to use alternatives to solve a problem; articulate mathematical explanations; demonstrate representations; Knowledge of Content and Students (KCS)-Knowledge that combines knowing about mathematics and knowing about students, Knowledge of how to anticipate what students are likely to think; relate mathematical ideas to developmentally appropriate language used by children; Knowledge of Content and Teaching (KCT): Knowledge that combines knowing about mathematics and knowing about teaching. In the present study we focussed on KCS which is a component of teachers' PCK (a subset of MKT).

Design and Methodology

Research Design

A mixed method involving qualitative (case study) and quantitative research design was used to identify one teacher's (Kim) emerging PCK as she participated in LS. Johnson and Christensen (2012) argued that such an approach was appropriate for classroom-based research. Data were collected through a video of Lesson Study teaching. Data was also acquired through a simulated recall interview with Kim, the teacher who is the focus of the study. During this interview, the video of the lesson was used to assist Kim to reflect on the events that happened during the teaching episode. Our analyses for this report are part of a larger study which involved the study of changes in mathematics and pedagogy of four teachers who chose different areas of primary school mathematics during the course of the LS. In the present study we focus on Kim's MKT as she engaged her children in the learning of fractions.

Participants. A group of six primary school teachers in the state of Penang (Malaysia) took part in a LS professional development program. The group was one of three groups of teachers in the school that took part in the program. The first group consists of teachers who were teaching Year 1 mathematics, the second group consists of teachers teaching Year 2 mathematics and the third group were teaching Year 3 mathematics. This paper reports on a teacher who was a member of the third group of teachers who were teaching Year 3 mathematics. Kim, was in her third year of teaching in the school. At the time of this study she was also undergoing a *Vacation Diploma in Teaching Course* (VDTC) to acquire teaching qualifications. Participants of the VDTC were regular teachers in the school but attended classes at the Teacher Education Institutes during the school vacation.

Lesson study as a professional development activity. The LS was conducted over a 6-month period during which all six teachers worked as a team in lesson design, implementation and reflection phase. There was an initial social visit to the school, the purpose of which was to introduce the idea of lesson study to the school teachers. Subsequently there were two more meetings with the teachers before the actual research lesson. During these meetings the teachers were grouped into three groups according to the level they were teaching (year 1, 2, and 3), for lesson planning.

The Year 3 teachers chose the topic on fractions for their lesson plan. The aim of the lesson was to introduce the concept of fractions. The participating teachers were invited to develop a series of lesson plans that were aimed at helping children adopt a problem-solving approach in mathematics. Kim and her team members wanted to develop a sense of problem-seeking and solving approach among her students.

The students were informed about the presence of teacher observers in the lesson. The class was the normal class where Kim was teaching the students for the past six months. The lesson was organised to engage the students using whole-class interaction for most parts of the lesson and small groups (five to six students in a group) during group activities.

The study examines MKT that was accessed by Kim during the course of her 40-minute lesson on fractions. The lesson was conducted in a Chinese primary school in the metropolitan area of Penang, Malaysia. Prior to the lesson observation, Kim had presented her lesson plans to her other five colleagues and invited their feedback. Prior to the lesson observation, the teachers met together with the external "knowledgeable others" twice. During the first meeting the Kim and her five other colleagues were introduced to the idea of lesson study. During the second meeting, the teachers together with the knowledgeable others began discussing the lesson to be observed. Kim volunteered to teach the lesson at the suggestion of the group. Subsequently, Kim and her colleagues in the lesson study group continued planning the lesson together.

Results

Structure of the Lesson

Kim's lesson can be described as consisting of three identifiable phases. In the *Introductory Phase* of the lesson, the teacher called two students to the front of the class and asked them to divide four sweets equally between them. This was a preamble to the lesson and it was mainly aimed at children's attention and involvement. The idea of dividing equally was an important concept and effective advanced organiser. This activity can be seen as providing an important clue for making connections between students' prior knowledge and a new concept that is to be processed and understood.

In the *Activity Phase* of the lesson, Kim used a combination of whole class teaching and group activity to introduce and develop the core concept of parts-whole and their relations. She showed that a fraction is one or more equally divided parts of a whole. In the first activity, the teacher displayed circles, cut into halves and quarters, on a flannel board. In the ensuing discourse, the teacher guided the class to say that the 'whole is cut into a number of equal parts' and 'the parts joined together makes one whole'. The activity was repeated using rectangles cut into four equal parts. While the first activity was driven by the teacher and therefore somewhat closed, in a subsequent activity, Kim recognised the need to provide space for the children to explore their understanding via an open-ended activity where the students work in groups to colour strips of papers into parts to show different fractions. The *Review Phase* of lesson involved Kim requesting students' views about the lesson and learning that was supported by the lesson. This information was obtained by students responding to a questionnaire.

Introductory Phase Episode

In the interaction below Kim was attempting to get the students to extend their understanding that the numbers of parts a whole can be divided into equal parts and that these parts put together make a whole.

Teacher:	One. Four over four is one, or what we discussed just now, two over two is one. This denotes one. Ok, now you look at the other set of building blocks. What can you notice? How many parts are there? What can you see?
Student:	Four parts.
Teacher:	Four parts. Ok, four parts. Other than seeing it as consisted of four parts, what else can you observe? Teacher: What is that?
Student:	Circle.
Teacher:	Circle. What else? What else can you see? Your..your building blocks. You look at your building blocks. What else can you see?
Student:	Four over four.
Teacher:	Four over four. Oh...any answer else? Any other answer? What else can you see? Yes, any other answer? This group? What else can you see? There are many…there are many…what is that?

We see evidence of the blending of content and emergence of PCK. The content is focussed on part-whole relations. The use of concrete aid (building blocks) to model the relations is indicative of Kim's knowledge of the use of multiple representations to support deeper understandings of the focus concept. Kim originally intended to use the idea of cutting into equal parts but the group suggested using cut pieces of equal size and having the students put the parts together. Kim found that group's idea was more practical and easy because the students would probably be not able to cut shapes into parts of equal size.

During this phase Kim also attempted to draw students' attention to the symbolic representation of fractions ('Four over four'). Comments such as "Any other answer? What else can you see? Yes, any other answer", suggest she was probing students and inviting responses that might provide the teacher with an insight into students' own meanings that were different from that of the teacher. We regard this action, again, as another case of PCK where a teacher shows empathy with her students and anticipates the construction of idiosyncratic meanings about fractions.

Kim explained that she did not want to tell students the answer, instead she would give some clues and let student try to answer. This belief was further reinforced by the lecturers in the VDTC, “My lecturer says that if you want students to learn, we must not give student answer straight away [so that] student will think.” For Kim, asking questions will encourage two-way communication between the teacher and the students thus avoiding one way communication which may lead the students to become bored.

Activity 1 Phase Episode

In this open-ended activity students were given paper strips and asked to fold and shade the fractions (See Figure 2). The students’ responses showed that they were able to fold the paper in many parts. Some students folded the paper strips into two parts while others were able to fold up to sixteen equal parts. Some parts were rectangular in shape while others were triangular.

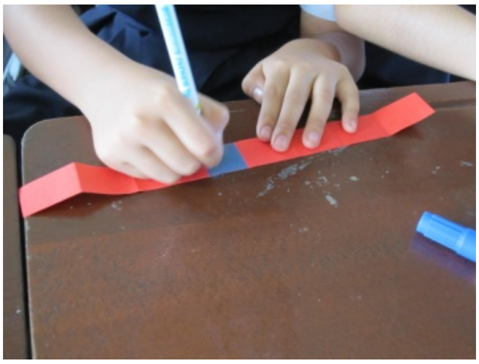

Figure 2. Folding and colouring paper strips into parts.

In the following interaction, Kim revisits her point about the relations between the number of parts and the whole in a fraction but this time with a model that involved strips of papers rather than circles. Her PCK in this instance indicates Kim’s understanding that modelling of part-whole relations should be not be fixed to any particular object.

Teacher: No, you don’t have to. Ok, a half. Ok, what else? … What else? (choosing paper strip) Xiang Yi divided her paper strip into four quarters. Ok, four parts: One, two, three, four. Ok, this is from Xiang Yi. Ok, Ethan’s… his is colourful. Ok. He divided..he divided it into how many parts? One, two, three, four, five, six, seven, eight. He divided it into eight parts. Ok, these are from the first group.

Teacher: How about the second group? The second group. Ok. Ok, let’s see Jing Rou’s. Jing Rou, see…let’s see Jing Rou’s. Jing Rou, raise your paper strip, turn around and let everyone see. Ok, how many parts did Jing Rou divide her paper strips into? How many? One, two, three, four, five, six, seven, eight. She made it into eight parts and coloured them. Any other else?

Student: (One of the group student) Mine is eight parts too.

Teacher: Oi, yours is eight parts too.

Student: (Another group student pointing to another group student) Teacher, he has extremely many parts.

Teacher: Hah, Puteri’s one is divided into four..four parts only. One, two, three, four. She divided it into four parts only.

Teacher: Who else? Do you have others?

Kim explained that she knew that students liked doing art, so if she could she could show the students how to separate the parts, they will be able to learn the idea of fractions.

Review Phase Episode

Table 1 provides responses from the students that was generated at the closure of Kim's lesson.

Table 1
Student Response about Classroom Environment

Statement	Yes (%)	No (%)
I shared with my classmates what I knew in the lesson	100	0
I got help from my classmates	88.5	11.5
I helped students who have trouble understanding the lesson	73.1	26.9
The teacher asked questions	88.5	11.5
I asked the teacher some questions	57.7	42.3
I asked my classmates some questions	65.4	34.6
My classmates talked with me about how to do the activities and problems	76.9	23.1
I showed and explained how I solved a problem to my classmates	76.9	23.1
I learned from my classmates in the lesson	88.5	11.5
The teacher was fair to me and my classmates	100	0
The Math I learned in the lesson can be used at home/the supermarket/store/everywhere.	88.5	11.5
I learned new and interesting things about Math in the lesson	84.6	15.4
What I learned is not useful at home/the supermarket/store/outside school.	76.9	23.1
I understood the lesson.	100	0
The lesson was fun.	96.2	3.8

From the student responses it is seen that most of the items received more than 70% of positive response. Two items, "I asked the teacher some questions" and "I asked my classmates some questions" scored 57.7% and 65.4% respectively.

As indicated in Table 1, the classroom discourse was guided mostly through teacher-initiated questions. It was however observed that Kim made an effort to restrain from giving the answers directly to her questions. Instead, often she probed the class through further questions. It was also observed that the teacher did not capitalise on the students various responses to further deepen the students' understanding of fractions.

Kim expressed her appreciation for being part of her LS group, "Normally you plan alone and you teach alone…[here we] Plan in group. When I am teaching, [I] need to improve. [Other] teachers can let me know [so] I will improve."

Discussion

We commenced our study on the premise that teachers' knowledge play a key role in the decisions they make about the content and delivery of their lessons. In so doing our main interest was the quality of one teacher's PCK that was accessed during her participation in a Lesson Study that was undertaken in a Malaysian Chinese primary school.

One aspect of this teacher's knowledge was her willingness to use a variety of activities in the classroom to engage students to experiment with different representations of the relations between parts and wholes. This engagement of students that was supported by the introduction of multiple representation gives one insight to her KCS (Knowledge of Content and Students), one of the three components of PCK are suggested by Ball et al (2008). One facet of Kim's KCS indicates that she had become aware of the different ways her students come to understand part-whole relations embedded in both symbolic and conceptual representations.

A second aspect of Kim's KCS can be related more to KCT, rather than KCS. We note the use of a blend of direct instructions and open-ended activities during the lesson. This is indicative of her view that students need clear direction about what to do and needed time to explore the connection between the notion of parts and wholes. On the basis of his work on cognitive processing of information, Mayer (2011:86) argued that teachers needed to provide learning experiences that not only promotes 'prime generative processing but also provide enough guidance to make sure that learners engage in appropriate amounts of essential processing and do not engage in excessive amount of extraneous processing'. We contend that the direct teaching phase of the lesson where Kim models part-whole relations as providing the required *essential processing* while her subsequent use of open-ended activities supporting *generative processing*. The latter activity was conducted in group situations thus allowing for reduction in cognitive load.

Kim's comments on the lesson strongly indicated that she placed high value on the practical aspects of PCK. For Kim, it was important to translate her knowledge into ways of organising student activities that were practical. This was also seen in her appreciation for her group members when they suggested practical and useful ways of managing the student activities.

Lewis, Perry and Murata (2006:13) argued that educational research needs to 'vigorously explore lesson study's potential as a tool of instructional improvement'. In the present study, we have undertaken such a study where a thicker description of a teacher's PCK during in-class action provides important directions about her own strengths and areas of knowledge that needs to be developed. In the long run, we argue that, this information can be expected have significant impact on the quality of mathematics instruction.

The description of Mathematics Knowledge for Teaching that is based on Lesson Study is an emerging field of research. In this study we aimed to capture one teachers' knowledge. It would be interesting to examine what effect this knowledge could have on that of the other teachers who participated in the LS. Equally important is to capture the knowledge of Chinese mathematics teachers that helps them orchestrate lessons that engage students more deeply than traditional classroom learning contexts.

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